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SHORT NOTES

Non-plane strain in section balancing: calculation of restoration parameters

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Abstract—We derive a set of equations which can be used to calculate the finite strain ellipsoid and longitudinal strains from measured strain axial ratios and orientations. The argument is based upon strain data from sections which carry the most relevant structural and kinematic data—the cross-section plane and the cleavage plane. These equations can be used to calculate pinline trajectories for correct restoration of sheared units in balanced sections.

INTRODUCTION

Many deformed geological sections show a more or less significant contribution by ductile strain and subscale distributed faulting (e.g. Woodward *et al.* 1986, Geiser 1988, Protzman & Mitra 1990, Dittmar *et al.* 1994). Ductile strain violates the traditional rules of section balancing because it implicates changes in line lengths, stratigraphic thickness or angles (e.g. fault cut-offs) and most possibly a change of the cross-section area. The construction techniques of balanced sections generally require that the section plane is strictly parallel to the particle displacement vector field of the deformed bodies. This requirement relates to the principle rule of area balance (Dahlstrom 1969), a geological conservation law which, in practice, amounts to the assumption of plane strain in the section plane. Thus a simple balance of line lengths meets the above plane strain requirement only in cases of no or weak distributed deformation. Several papers have so far dealt with the geometrical consequences of strain (e.g. Cobbold 1979, Ramsay & Huber 1983, Woodward *et al.* 1989, Howard 1993) and propose strategies for including strain data in section restoration, while however maintaining the rule of area balance or plane strain.

In many natural cases, the plane strain rule is not strictly realised. Either strain is not plane, and/or the principal plane of strain (XZ) does not parallel the section plane (the XZ plane usually is perpendicular to the strike of major structures and contains the direction of assumed tectonic transport), or area is not conserved due to volume change. In these cases, especially in the more realistic case of the non-coaxial superimposition of strain and displacement increments, the fundamental requirement for section balancing is violated and restoration of sections in two dimensions does not yield a strictly balanced solution. Application of balancing techniques which were originally designed for external parts of fold belts to the more internal parts by several workers has proved quite successful in understanding the internal geometry and kinematic evolution of internally strained belts (Dittmar *et al.* 1994). In these more weakly constrained situations some correction has to be performed on bed lengths, bed thicknesses, cross-section area and restoration procedures which are based on assumptions or observations of the geometrical deformation mechanism (i.e. flexural slip, simple shear, etc.).

The present paper develops a set of equations which can be used to calculate the finite strain ellipsoid and longitudinal strains from measured strain axial ratios and orientations as well as to calculate pinline trajectories for correct restoration of sheared units. The argument is based on easily available standard strain data from sections which carry the most relevant structural and kinematic information (namely cross-section and cleavage plane).

CALCULATION OF AXIAL ELONGATIONS

The three-dimensional state of strain is expressed by the ellipticities of the three principal planes of strain and is usually written in terms of the ratios of the principal extensions ($R_{XZ} = (1 + e_1)/(1 + e_3)$, $R_{YZ} = (1 + e_2)/(1 + e_3)$, and $R_{XY} = (1 + e_1)/(1 + e_2)$).

The volume V_E of an ellipsoid with $V_E = V_{US}$, volume of the unit sphere, is $V_E = 4/3\,\pi * a * b * c$ where a, b and c are semiaxes of the ellipsoid. The volumetric dilation of the strain ellipsoid $1 + dV$ is

$$1 + dV = a * b * c = (1 + e_1) * (1 + e_2) * (1 + e_3). \qquad (1)$$

From the final and the original volume dV is defined as $(V_f - V_O)/V_O$. Equation (1) can be rearranged to derive $(1 + e_1)$ from the principal axial ratios

$$(1 + e_1) = ((R^2_{XZ} * (1 + dV))/R_{YZ})^{1/3} \quad (2)$$

and for the intermediate and short axes:

$$(1 + e_2) = ((R_{XZ} * (1 + dV))/R^2_{XY}{}^{1})^{1/3} \quad (3)$$

$$(1 + e_3) = ((R_{XY} * (1 + dV))/R^2_{XZ})^{1/3}. \quad (4)$$

Changes of length in any other direction within the strain ellipsoid can be calculated if their orientation with respect to the principal extensions and their axial ratios is known.

CALCULATION OF LONGITUDINAL STRAINS

Measured angles between the line whose longitudinal strain is to be determined and the ellipse major axis can be used in the equations for the ellipse or the ellipsoid in order to determine the longitudinal strain of any line. The Cartesian equation of an ellipse is (Fig. 1a):

$$x'^2/a^2 + y'^2/b^2 = 1. \quad (5)$$

The coordinates of a point P′ on the ellipse are

$$x' = r' * \cos\alpha' \quad (6)$$

$$y' = r' * \sin\alpha' \quad (7)$$

(with x' and y': coordinates of an ellipse point P′, r' : radius of P′, and α': angle between the ellipse major axis and OP′). Inserting (6) and (7) in (5) gives:

$$(r' * \cos\alpha')^2/a^2 + r' * \sin\alpha')^2/b^2 = 1.$$

Isolation of r' yields:

$$r' = 1/(\cos^2\alpha'/a^2 + \sin^2\alpha'/b^2)^{1/2}. \quad (8)$$

The equivalent percent extension is:

$$dL[\%] = (r' - 1) * 100. \quad (9)$$

The three-dimensional case follows an analogous procedure. The Cartesian equation for the strain ellipsoid is:

$$x'^2/a^2 + y'^2/b^2 + z'^2/c^2 = 1. \quad (10)$$

The trigonometric relationships in an ellipsoid yield the following equations for the coordinates of P′ (with r'', the radius of the projection of P′ onto the XY-plane (see Fig. 1b) replaced by $r'' = r' * \cos\beta'$):

$$x' = \cos\alpha' * r' * \cos\beta' \quad (11)$$

$$y' = \sin\alpha' * r' * \cos\beta' \quad (12)$$

$$z' = \sin\beta' * r' \quad (13)$$

[r': radius of point (P′) of the ellipsoid shell; see Fig. 1b]. These equations are inserted in (10) which is then rearranged to give r'. In this case—because reference is taken to the principal axes of the strain ellipsoid—a, b and c can be replaced by the principal extensions.

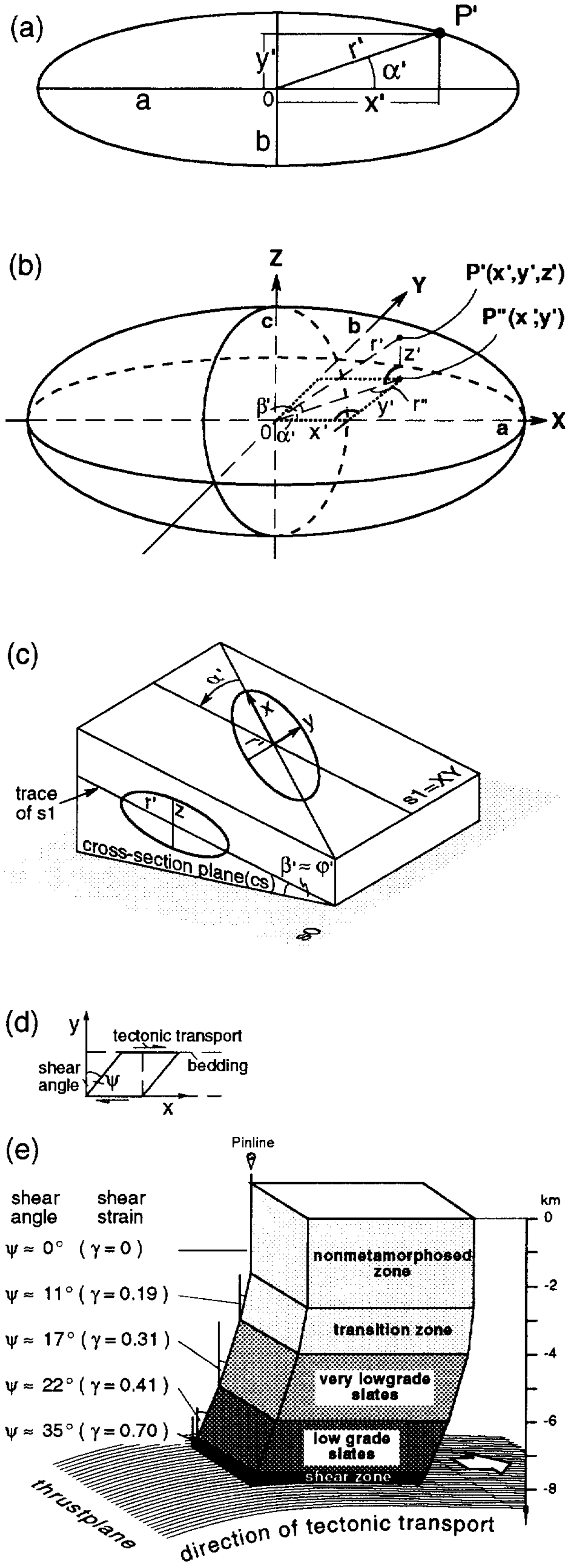

Fig. 1. Geometries for the calculation of longitudinal strains within (a) the two-dimensional strain ellipse, (b) the three-dimensional strain ellipsoid, and (c) for the calculation of the three-dimensional state of strain from one principal and one oblique section (s_1—cleavage, s_0—bedding). (d) Bedding parallel shear of material lines originally perpendicular to bedding; (e) example of a synthetic pinline trace for a thick, pervasively deformed and sheared thrust body from the Rhenish Massif, Mid-European Variscides (see Dittmar *et al.* 1994).

$$r' = ((\cos\alpha' * \cos\beta')^2/(1+e_1)^2 + (\sin\alpha' * \cos\beta')^2/(1+e_2)^2 + (\sin\beta')^2/(1+e_3)^2)^{-1/2}. \quad (14)$$

As in the two-dimensional case the percent longitudinal strain along the chosen line can be expressed using equation (9). In section balancing the reciprocal value of r' can be used for correcting bedlength or the stratigraphic thickness (see Ramsay & Huber 1983) if α' and β' are chosen appropriately. The factor takes into account the influence of dilatation if the three-dimensional state of strain was calculated as shown in (2) – (4). At the same time, longitudinal strains can be calculated for any direction required, namely changes of length parallel and perpendicular to the section.

CALCULATION OF THE STRAIN ELLIPSOID FROM ONE PRINCIPAL PLANE OF STRAIN AND ONE OBLIQUE SECTION

Three-dimensional measurements usually are calculated from the axial ratios of at least two principal sections of the strain ellipsoid or from three arbitrary sections. From an economical point of view it is usually found convenient to focus analysis on two sections. Field practice in an accompanying study (Dittmar *et al.* 1994) has shown that the principal planes are not always accurately defined by fabrics or they may be oblique to the cross-section plane defined by field structural data. The cleavage plane mostly approximates the XY-plane of the finite ellipsoid, but the long ellipse axis may show a high orientation variability in the normally investigated cleavage plane and is mostly oblique to fold axes. In section balancing the second section to investigate usually is the cross-section plane. Both sections can be used to calculate the entire finite strain ellipsoid because its orientation is confined by the above observations.

The calculation of the three-dimensional strain from strain measurements in two sections, only one being a principal plane of strain, uses the following procedure (see Fig. 1c). The data required are:

- One axial ratio (R) of a principal plane of strain, generally the R_{XY}-value;
- the angle (α') between the major axis of the ellipse in this principal plane and the orientation of the second plane which is subperpendicular to the first plane;
- the axial ratio of the strain ellipse within the second plane (i.e. the cross-section plane, R_{CS}).

$$R_{CS} = r'/(1+e_3). \quad (15)$$

This equation is rearranged using (8) for the XY-plane:

$$(1+e_3) = (\cos^2\alpha'/(1+e_1)^2 + \sin^2\alpha'/(1+e_2)^2)^{-1/2} * R_{CS}^{-1} \quad (16)$$

$(1+e_2)$ is replaced using $R_{XY} = (1+e_1)/(1+e_2)$ which changes (16) to

$$(1+e_3) = (1+e_1)/(R_{CS} * (\cos^2\alpha' + (\sin^2\alpha' * R_{XY}^2))^{1/2}). \quad (17)$$

The axial ratio of a second principal plane of strain, R_{XZ}, may now be calculated using the expression in (17):

$$R_{XZ} = R_{CS} * (\cos^2\alpha' + (\sin^2\alpha' * R_{XY}^2))^{-1/2}. \quad (18)$$

The third ellipsoid plane is calculated from $R_{YZ} = R_{XZ}/R_{XY}$ which then allows determination of the ellipsoid shape factor and of the longitudinal strains along the principal strain axes [equations (2)–(4)].

CALCULATION OF STRAIN-CORRECTED PINLINE GEOMETRIES

Like flexural slip or flexural flow, pervasive deformation such as the formation of slaty cleavage or fault-related bed parallel shear exerts a component of simple shear parallel to the planes of bedding. Bedding planes are usually taken as the marker lines for restoration (Fig. 1d). Material lines which were originally oriented perpendicular to bedding will usually not maintain this position due to this shear. Pinlines, representing such material lines, are placed as a rule where no finite bedding parallel shear during flexural slip is assumed to have accumulated: in the undeformed foreland, at axial planes of major symmetrical folds, at crestal planes of ramp anticlines, at the 'return to regional'. In the last case and in those cases where positioning of a local pinline on a ramp can not be avoided due to the lack of other possibilities within a thrust body, their geometry has to be corrected. Without this procedure restoration of angles to their predeformational stage (i.e. cut-off angles etc.) may show distortion and sections may not balance.

Bedding parallel shear in the section plane by ductile deformation depends on the strain in cross-section (R_{CS}) and the angle ϕ' between strain axes and bedding in cross-section (Fig. 1c, see Ramsay & Huber 1983)

$$\Psi = \arctan(((R_{CS}^2 - 1) * \tan\phi')/((1 + R_{CS}^2) * \tan^2\phi')). \quad (19)$$

The angle obtained is used to draw a pinline deviating from the orientation perpendicular to bedding in the direction of local shear. Eventual slip on bedding planes adds to this shear. Generally a complete pinline profile through the lithologies of a thrust body has to be calculated from strain data (Fig. 1e). A free program calculating the presented restoration parameters is available from the first author, if a 3.5″-disc is provided.

DISCUSSION AND CONCLUSIONS

The procedures presented so far are aimed at properly restoring deformed single thrust sheets. The restoration of displacement of the latter on faults into or out of the section requires further information. Only in the cases where the lateral structural continuity of the structures modelled is definitely larger than the inferred out of section displacement, can it be assumed that the effects on restoration are negligible. In practice, restoration of deformed units has to take into account a number of further aspects, some of which are outlined.

The equations clearly show that for non-plane strain deformation information on orientation and shape of the finite strain ellipsoid—i.e. by three-dimensional strain analysis—as well as volumetric dilatation is needed to properly determine longitudinal strains. Due to fundamental problems in determining volumetric changes the latter is neglected in most cases. Generally, this will result in an underestimate of shortening and basin width (cf. Mitra 1994). Moreover, the term for volumetric dilation in the equations does not distinguish its probable anisotropic effects on the three principal stretches e.g. in the case of pressure solution or breakdown of pore volume in an open system where volumetric strain may be completely partitioned in shortening one axis while leaving the others unaffected [dV in equations (3) and (4) will be zero in these cases]. Only small scale redistribution of matter in a system closed at least on thin section scale will be detected by standard strain analysis techniques (e.g. centre to centre and related techniques) but will not quantitatively resolve the partitioning between the different active shape changing mechanisms. Since the complete volumetric dilatation tensor is not usually known, this simplification has to be accepted in most cases.

Complete restoration of deformed thrust units including finite strain is not a straightforward procedure because superposition of strain increments is non-commutative. Ideally therefore, the identified sequence of increments should be restored sequentially (cf. Cobbold 1979, Protzmann & Mitra 1990). On average, a sequence of compactional and one or more non-coaxial tectonic strain increments will have occurred. Identification and quantification of these will not usually be unequivocal, especially with relationship to rotation of and folding within thrust units. Simplified practical solutions will therefore tend to restore displacement and deformation of a thrust body in a single step (Howard 1993). The error introduced by this procedure is however estimated to be well below that caused by complete neglect of strain data. Restoration based on the above type of data, whether single or multi-step, is suggested to proceed from marker lines (pinlines, bedding, etc.) which secures internal coherency and strain compatibility (cf. Ramsay & Huber 1983). This procedure is in contrast to other strain restoration techniques which restore either homogeneously deformed segments along calculated displacement vectors (Howard 1993) or apply a finite element technique (Cobbold 1979, Woodward *et al.* 1986).

It is obvious from the above discussed problems that advances in balancing techniques which claim to reproduce the complexities of nature not only need to incorporate strain data but have to include three-dimensional volume construction and restoration. In practice the data obtained from surface sampling within a defined structure have to be used to infer bed length correction factors as well as synthetic pinline traces for a complete thrust body which is not entirely satisfactory. Because strain in general is heterogeneous, ideally, a complete mapping of three-dimensional strain in a thrust body is required. Due to the impracticability of this demand, techniques of extrapolation or prediction of strain are needed to approximate natural strain distributions. Apart from forward modelling techniques, the systematic relationships between finite strain and controlling parameters like grain size, composition, structural position and deformation temperature, found for example by Dittmar *et al.* (1994) may show the way.

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